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Vibration characteristics of vaulted masonry monuments undergoing differential support settlement

Sezer Atamturktur [a,*], Luke Bornn [b,1], François Hemez [c,2]

[a] Department of Civil Engineering, Clemson University, Lowry Hall, Clemson, SC, 29634, United States
[b] Department of Statistics, University of British Columbia, 333-6356 Agricultural Rd. Vancouver, BC V6T 1Z2 Canada
[c] XTD-3, Primary Physics, Los Alamos National Laboratory, Mail Stop T087, Los Alamos, NM 87545, United States

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ABSTRACT

This paper assesses the feasibility of vibration testing to detect structural damage caused by the settlement of buttresses in the Beverley Minster, a Gothic church located in the UK. Over the past eight centuries, the accumulated support settlements of the buttresses of Beverley Minster have pulled the main nave walls outward, causing severe separation along the edges of the masonry vaults. Bays closer to the main crossing tower have remained intact; however, at the west end of the Minster, the crack width between the walls and vaults has reached about 150 mm, leading to approximately 200 mm of sag at the crown of the vaults. Due to uneven settlement of buttresses along the nave of the church, the Minster now has ten nominally identical vaults at different damage states. In this work, two of these vaults representing the two extremes, the most damaged and undamaged structural states, are subjected to vibration testing with impact hammer excitation. From these vibration measurements, damage indicators are extracted in the modal, frequency, and time domains. In the modal domain, the differences between modal parameters are observed to be comparable to measurement uncertainty and hence insufficient to reach conclusions about the presence of vault damage. However, the amplitudes of frequency response functions in the frequency domain are observed to indicate a clear difference between the damaged and undamaged states of the structure. A time domain autoregressive model, support vector machine regression, is also found to be successful at indicating the differences between the two structural states of the vaults. We conclude that vibration measurements offer a practical solution to detect wall–vault separation in historic masonry monuments, provided that multiple damage indicators are evaluated.

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1. Introduction

Masonry is a common building material in many historic monuments and has unique intrinsic properties that make it particularly susceptible to differential support settlements. Support settlement is a more frequent problem among masonry buildings because masonry structural systems tend to be significantly heavier than those of reinforced concrete or steel buildings. When the demand for large bearing capacities from supporting foundations are not met due to deteriorating soil conditions, the supports of a masonry building incrementally settle and induce tensile forces in the structure. However, unreinforced masonry buildings are primarily designed to be loaded in compression; as such, they are characterized by stiff units separated by relatively soft mortar joints. As a result, tensile forces induced by differential support settlement easily lead to geometric distortion and structural discontinuity, which alter the mass, stiffness and energy dissipation properties of the structure. Since the vibration response is intimately dependent on these properties, the change in the structural behavior due to damage may be detectable by vibration measurements. This hypothesis is the focal point of this manuscript.

The success of vibration-testing-based structural health monitoring (SHM) depends not only on the structural characteristics of the building and the type and severity of damage, but also on the response *features* used to characterize the vibration properties. In an ideal situation, a measured vibration response feature is directly correlated to the presence and extent of damage. However, in practice the response of a structure is typically measured in terms of time-dependent acceleration. Any attempt to directly correlate these raw time domain acceleration measurements to structural damage is hindered by the sensitivity of the time domain response to many factors, such as environmental conditions and ambient vibrations that are unrelated to the presence or extent of damage. Therefore, data processing and/or coordinate transformation

* Corresponding author. Tel.: +1 864 656 3003; fax: +1 864 656 2670.
E-mail addresses: sez@clemson.edu (S. Atamturktur), l.bornn@stat.ubc.ca (L. Born), hemez@lanl.gov (F. Hemez).
1 Tel.: +1 604 822 0570.
2 Tel.: +1 505 667 4631.

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Fig. 1. The interior view of the nave of the Beverley Minster displays the limestone piers that support the stone vaulting. The leaning of the columns outwards is visually observable on site.

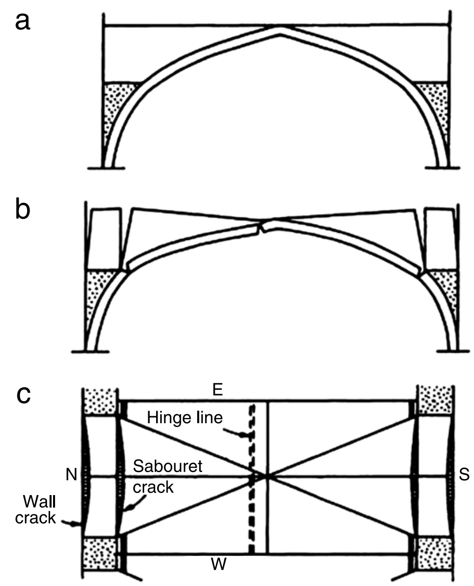

Fig. 2. A schematic of Sabouret Cracks, by Heyman [2] (with permission).

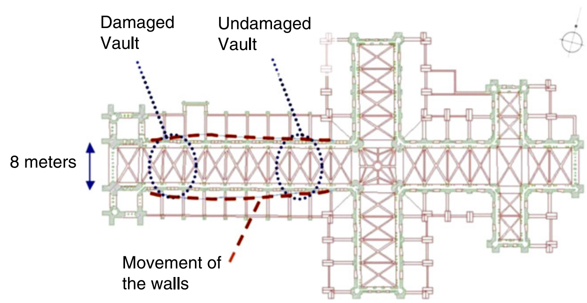

Fig. 3. The movement of the walls is not uniform along the length of the nave; as such the Minster now has ten vaults with varying damage states. Two vaults representing the most damaged and undamaged states are selected for the study.

become necessary to extract low-dimensional diagnostic features from the raw time domain measurements. The clear requirement for these features is that they must be sensitive to damage and insensitive to noise factors, such as changes in environmental conditions and ambient vibrations. This requirement makes feature selection challenging since both the damage-sensitivity and noise-sensitivity of vibration features are application-specific. Therefore, the most suitable feature for a structure with a particular type of damage may be unsuitable for another structure or even for a different type of damage within the same structure. As a result, the damage-sensitivity of vibration features for a given structural system must be individually evaluated for a given damage scenario. In this manuscript, we evaluate the damage-sensitivity of various vibration response features to the separation between walls and vaults, a common structural problem in Gothic churches.

This evaluation can be performed most effectively by separately testing damaged and undamaged states of the same structure. However, one can hardly imagine damaging an existing historic structure for such evaluations. In fact, engineers involved in SHM applications rarely have the opportunity to test an existing structure in its damaged and undamaged states. Considering this difficulty, Beverley Minster presents a unique opportunity by allowing the investigation of ten masonry vaults, which are substantially similar in their geometry, boundary conditions, construction materials, erection technique and workmanship, varying only in the extent of structural damage they have endured (Fig. 1). Structural damage in Beverley Minster's vaults manifests itself primarily as Sabouret cracks [1] and has been primarily caused by settlement of nave buttress foundations (Fig. 2). Section 3 discusses the details of the damage in Beverley Minster's vaults and briefly overviews the history of the structure.

In the present study, two vaults, one which exhibits the most severe wall–vault separation and the other visually no wall–vault separation, are selected and subjected to vibration testing. Hereafter, these two vaults are referred to as the damaged and undamaged vault prototypes (Fig. 3). These two prototypes provide the opportunity to obtain vibration measurements from two different structural states of otherwise similar vaults of Beverley Minster. With this statement comes a caveat; these two prototypes are assumed to be different only in their damage states, while their initial geometry, boundary conditions, construction materials, erection techniques and workmanship are accepted to be sufficiently similar. Actions taken to justify this assumption include (1) performing full-size geometrical surveys to determine geometric variability, (2) conducting local non-destructive tests to estimate material variability and finally (3) simulating the effect that estimated geometric and material variability have on the vibration response of the structure through finite element models. Section 4 discusses the actions taken to quantify the vault-to-vault variability and Section 5 discusses the finite element model simulations. The finite element model simulations illustrate that the vault-to-vault variations have an insignificant effect relative to the effect of the structural damage on the vibration response.

Section 6 overviews the adopted testing campaign, namely, vibration testing with an impact hammer. The following sections discuss the evaluation of the collected vibration measurements. In Section 7, this evaluation is completed in the modal domain. Section 7 includes the finding that certain modal features, such as natural frequencies and mode shapes of the first three modes, fail to indicate the differences in the structural states of the two prototype vaults. Further evaluations are completed in the frequency and time domains in Sections 8 and 9, respectively. In the frequency domain, the amplitudes of frequency response functions (FRF) acquired from the damaged vault are noticeably higher than that of the undamaged vault, providing a clear indication of the structural differences between the two vaults. In the time domain, the time domain autoregressive methodology implemented herein also exhibits sensitivity to the damage present in the vaults and provides clear indication of the structural differences between the two vaults.

2. Background

Although the majority of vibration-based damage detection studies in civil engineering literature focuses on reinforced concrete and steel structures, a number of key studies on historic masonry monuments have been reported over the past three decades. In this section, we organize these relevant studies into three categories: scaled laboratory models, existing damaged structures, and structures with retrofit.

Scaled laboratory models: One of the earlier studies on vibration-based damage detection was applied by Armstrong et al. [3] to investigate spandrel wall separation. The authors measured the vibration response of two scaled masonry arch bridge models under impact hammer excitation, one of which featured damage due to wall separation. The authors had success in relating the deviations in modal parameters obtained from the two scaled arch bridge models to their structural condition. Armstrong et al. [4] obtained consistent results when a similar study was performed on the scaled arch bridge models that focused on the dynamic stiffness instead of modal parameters. These two successful studies suggest that vibration measurements are a viable monitoring tool for detecting spandrel wall separation in existing masonry arch bridges. However, the authors stressed the necessity to investigate the damage-sensitivity of arch bridge vibration characteristics to a wider range of structural defects. This call to evaluate the damage-sensitivity of vibration characteristics for a wider range of damage scenarios is reflected in the attempt of the present study to investigate the feasibility of vibration testing to detect damage induced by support settlement.

Bensalem et al. [5,6] also investigated the vibration response of scaled brick arch models. By observing the difference in the peak amplitude frequency response functions, Bensalem et al. [7] detected void presence and size in the arch bridge backfill. This finding is consistent with the observations of the present study; see Section 8.

SHM tools have also been applied to scaled masonry building models. Vestroni et al. [8] tested a 1/5th scale masonry building under shaker excitation. Vestroni et al. [8] successively increased the excitation force and incrementally induced structural damage. As damage was induced, a reduction in the dynamic stiffness was observed. Ramos [9] also had success in observing a consistent decrease in natural frequencies as the cracks in a full-scale rubble stone building successively increased. Ramos [10] conducted a similar study on scaled arch and wall models built with clay bricks of low compressive strength and mortar with poor mechanical properties, such that the models were representative of historic construction. Controlled static forces were applied to the scaled models to progressively induce cracks. By monitoring the modal parameters of the scaled models, a clear loss of stiffness was observed after the first crack. Modal parameters provided evidence consistent with damage in the system – with increasing levels of damage, frequencies were reduced while damping coefficients were increased. In contrast with the natural frequencies, Ramos [10] noted that the mode shapes of the test structure generally remained unchanged.

Existing damaged structures: Studies conducted under controlled laboratory conditions are largely immune from complications caused by support settlements, environmental loads, material deterioration, prior damage, and operational conditions. That is why laboratory experiments on scaled masonry models typically yield higher quality measurements compared to the tests conducted on existing masonry structures. Moreover, laboratory experiments often overlook the practical difficulties of performing in situ vibration tests, thus providing a poor reflection of the difficulties involved in SHM. Therefore, studies completed on existing structures are of great value for SHM literature.

Gentile and Saisi [11] completed a damage detection study on a historic masonry tower based on finite element model calibration. The tower was partially damaged with extensive vertical cracks as a result of excessive compressive forces. The modal parameters of the tower were identified by ambient vibration testing. The finite element model was built with six distinct regions each representing a different damage severity. After calibration, the finite element model yielded relatively low Young's modulus values in the damaged regions, illustrating the potential of simulation-based methods to deliver useful information about the state and location of damage in a masonry structure. A unique vibration-based damage detection study was completed on 534 stone pinnacles in the Palace of Westminster in London [12]. The natural frequencies of the pinnacles were obtained by using impact excitation for smaller pinnacles and exploiting wind excitation for larger pinnacles. The natural frequencies of the pinnacles were compared against each other and the outliers were detected. The five pinnacles with outlier natural frequencies were successfully identified as damaged pinnacles.

As seen when investigating an existing structure, the analysts can typically collect only a restricted number of measurements from either one of the undamaged or damaged states. This restriction has received significant attention in technical literature. Some researchers attempted to simulate the damage scenarios with numerical models [13], while others focus on scaled experimental specimens at undamaged vs. damaged states [14]. The former approach is hindered by errors and uncertainty inherent in the numerical simulations, while the latter fails to represent the challenges present in real life applications. Methods successful in detecting damage, in the absence of *a priori* data from the undamaged structure, focus on outliers and novelty analysis to detect the onset of future damage. These methods have recently been deployed on historic masonry monuments [15,16]. Implicit in this approach is the assumption that damage will manifest itself as observable changes in the vibration measurements [17]. One contribution of the present manuscript is to demonstrate that this assumption is not always applicable.

Structures with retrofit: An alternative approach to gain information about the various states of a structural system is the assessment of structural improvements after retrofit or strengthening campaigns. In their ambient vibration analysis, which compared identified modal parameters before and after retrofit, Turek et al. [18] found an increase in the dynamic structural stiffness of a recently repaired historic church. Increased dynamic stiffness after retrofit was also observed in a similar study on a historic basilica by Antonacci et al. [19] and on a historic masonry tower by Ramos [10].

The previously successful studies, whether focusing on scaled laboratory models or existing structures, predominantly use differences in modal parameters or their derivatives as damage indicators. In earlier studies, other response features, such as frequency domain or time domain features, have seldom been incorporated. Furthermore, there is a need to investigate the damage-sensitivity of vibration response features for a variety of masonry structures, e.g. arch bridges, towers and churches, under all plausible damage scenarios. Another contribution of this work is to take a step in this direction by proposing response features that may be better indicators of wall–vault separation in Gothic churches.

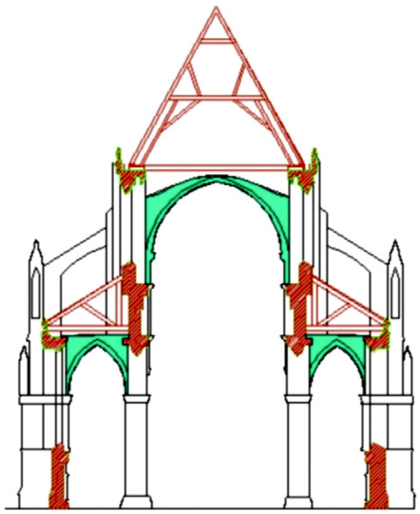

Fig. 4. The cross section of the vault displays how the stone walls and buttresses horizontally support the nave vaults. The settlement of the buttresses pulls the walls outward, causing separation between the walls and vaults.

3. Beverley Minster

Beverley Minster is typical of Gothic churches. The vaulted ceilings of the Minster are supported vertically by stone arches and piers and horizontally by flying buttresses. The church is predominantly constructed with limestone blocks of varying strengths and brickwork, which forms the vaults [20] (Fig. 4).

According to historical documents, the movement of the nave walls has been a concern since its construction in the early thirteenth century. Ever since the nave walls were erected, they have increasingly leaned out due to the foundation settlement of the buttresses [21]. In the eighteenth century, ties were added at roof level to prevent further separation of the walls. According to historical documents, the outward movement progressed and a century later wood beams spanning the width of the nave were fixed to the nave walls with steel ties in the hope of mitigating further deformation. Although this intervention was partially effective, it did not completely eliminate movement in the nave walls [22].

The settlement of the buttresses has pulled the nave walls outward, detaching the walls from the masonry vaults. With this separation, vaults have been unable to transfer horizontal thrust to the walls, resulting in the flattening of the nave vaults. The movement of walls, however, has not been uniform along the length of the nave. Assessment reports, completed by Price & Meyers Consulting Engineers in 2004, document the magnitude and patterns of wall movement. According to the site survey, maximum separation between the walls and vaults of 135 mm (5.3'') occurs at the west side of the nave (Fig. 5). The vaults at the east end of the nave, however, appear restrained by the tower and the walls and buttresses of the transepts, thus remaining intact. As a result, Beverley Minster, in its current state, has ten vaults with different damage states [22].

As seen in Fig. 5, while there is a gap between the walls and vaults for the damaged vaults, the brick webbing of the undamaged vault rests intact on the stonewalls. However, the degree of lateral restraint provided to the undamaged vaults by the nave walls is difficult to determine. During the finite element simulations, discussed in Section 5, this difference in the two vaults is represented as free lateral movement and restrained lateral movement.

Fig. 5. The interface between the nave walls and vaults of (left) damaged and (right) undamaged vaults. The gap between the walls and vaults of the undamaged vault is filled with plastic sheets.

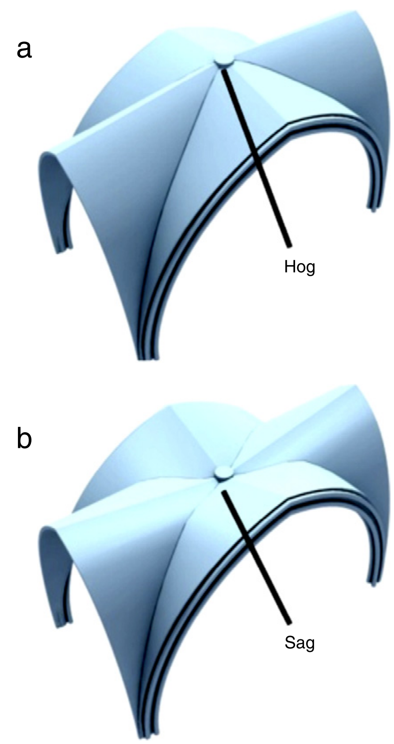

Fig. 6. The originally concave down curvature of the vaults is flattened: (top) undamaged and (bottom) damaged vault. The formation of the 6 ''wide Sabouret cracks results in an 8'' sagging of the crown of the vaults.

4. Vaulted structure under study

The two prototype vaults investigated in this study have two significant differences due to damage: the Sabouret cracks and consequent geometric distortion (Figs. 2 and 6). Aside from these two aspects, the two prototype vaults are expected to exhibit minor variations in their geometric and material properties. The present study relies on the important premise that these minor differences between the two prototype vaults have negligible effect on the vibration response. This premise will later be verified through FE simulations in Section 5. In the present section, however, we discuss two campaigns implemented to quantify

(1) variations in the vault geometry and (2) the natural variations in the construction material. The findings of these two campaigns are then entered into the finite element models to quantify the effect that geometric and material variability have on the vault vibration response.

4.1. Vault-to-vault geometric variability due to construction imperfections

The vault-to-vault geometric variability discussed herein is the variability due to inconsistencies associated with the medieval construction techniques of erecting masonry vaults. These inconsistencies are expected to result in slight geometric deviations among the ten vaults, even before settlement of buttresses occurs. To estimate the degree of this geometric imperfection, a three-dimensional survey of both the vaults are completed using a Leica TPS800 series survey instrument. The upper surface (extrados) and the lower surface (intrados) of the vaults are surveyed. The survey points are taken primarily at the crowns, along the ribs, around the surcharge, and along the edge of the nave walls.

Comparison of the three-dimensional geometry of undamaged and damaged vaults should yield the geometric variability due to the combined effects of imperfect construction and structural damage. However, because the intention herein is to estimate the inherent geometric variability in construction prior to the occurrence of damage, the symmetric design of the vaults is exploited. By calculating the deviations between the four quarters of the undamaged vault, the maximum geometric variability in one vault is estimated to be 6%. The geometric variability is estimated to be predominant along the longitudinal direction of the nave. In Section 5, this variability will be represented by increasing the longitudinal dimension of the vaults in the FE model by 6%.

4.2. Vault-to-vault variability due to materials

The Impact Echo (IE) method is conceptually based on the fact that the waves propagating through the thickness of a material are reflected when they encounter a change in medium [23]. Due to the larger wavelengths (typically greater than 10 cm) required for the IE method, wave diffusion through aggregates, cracks, and pores has less degrading effects than in ultrasonic testing [24,25]. Therefore, IE provides a viable solution for non-destructive testing of masonry assemblies.

In this study, IE tests are conducted to estimate the natural variability of brick units and mortar assembly. A total of 30 tests are conducted at various locations on the vaults. During the tests, the vault webbing is impacted by a hardened steel ball and the localized, high-frequency vibrations caused by this impact are measured through a displacement transducer. The main resonant frequency of stress wave reflections between the internal and external boundaries of the masonry vault webbing is captured. Fig. 7 illustrates a select few of these measurements. From the dominant frequency, the overall time required for a single cycle is obtained. Assuming that the vault thickness remains constant, the material properties of the vaults are mathematically related to the velocity of stress waves. The variation in the ratio of Young's modulus over density of the masonry assembly is estimated to be roughly 10%.

5. Finite element simulations of the vaults

In the previous section, uncontrolled variations in the geometric and material properties of the nave vaults were approximated through geometric surveying and impact echo testing. In this section, the effects of these variations on the vibration response are investigated. This is necessary to verify that the change in the selected features due to damage is differentiable from that due to vault-to-vault variability. First, the development of the FE model is discussed, and then the methods used to estimate the variability in geometric and material properties are introduced. Finally, the post-processing of the time domain simulations is addressed.

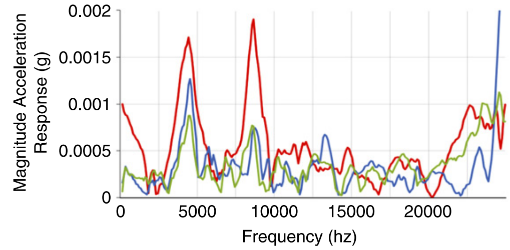

Fig. 7. The impact echo measurements are used to estimate the standard deviation of the homogenized material properties of the mortar and masonry assembly.

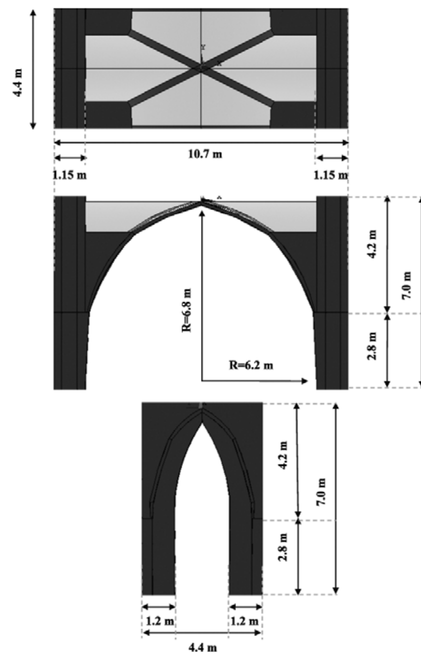

Fig. 8. The dimensions of the masonry vaults of the Beverley Minster.

5.1. Development of the finite element model

The primary purpose of the finite element model is not to make predictions about the structural behavior of the vaults at Beverley Minster, nor to reproduce the experimental measurements. Instead, it is used to investigate the noise-sensitivity of the vibration response features, which is the sensitivity to variations in geometric and material properties.

The initial steps in the development of a three-dimensional finite element model are the reproduction and simplification of the three-dimensional geometry. During this step, both the drawings by Price & Meyers Consulting Engineers and the measurements from the *onsite* three-dimensional survey are used (Fig. 8). The next step consists of creating a solid model utilizing the commercially available software ANSYS v. 10. Solid modeling is followed by mimicking the material properties and boundary conditions of Beverley Minster in the model (Fig. 9). The material

Table 1
Prior knowledge on the material properties of structural components.

Component	Material type	Modulus of elasticity (E)		Density (d)
		Low (GPa)	High (GPa)	Nominal (kg/m^3)
Walls, columns, vault ribs	Indiana limestone and Type O mortar	6	28	2100
Vault webbing	Brick	1	6	2100
Fill	Rubble and earth	0.5	5	2100

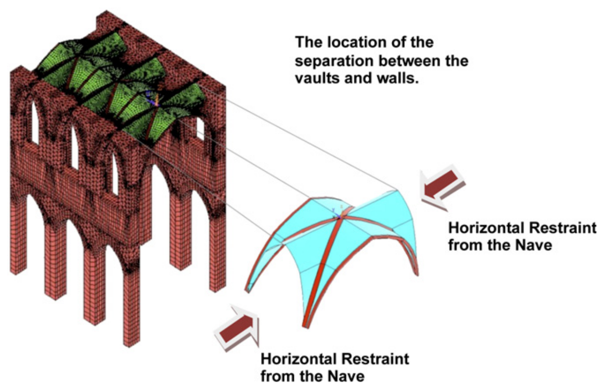

Fig. 9. FE model of Beverley Minster used to simulate the changes in the vibration response of the vaults due to the variability in geometry and material properties present in the Minster.

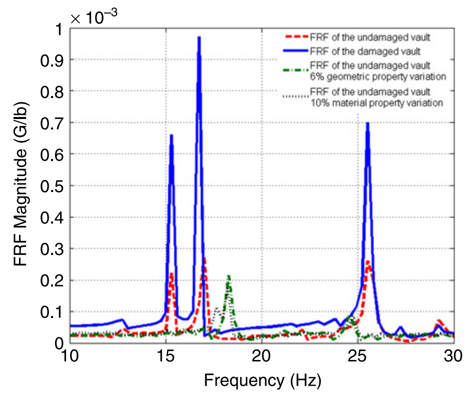

Fig. 10. The FRFs are simulated for the four scenarios: (1) the undamaged vault, (2) the damaged vault, (3) the undamaged vault with variation in geometry and (4) the undamaged vault with variation in material.

behavior of masonry is simulated using a linear-elastic constitutive law. The material properties, such as Young's modulus and density, are selected according to a review of pertinent literature (Table 1). These parameters are calibrated by comparing the ANSYS simulation results with experimental measurements [26]. The finite element model is conceived to simulate the undamped vibrations of the vaults, and thus damping factors are not defined.

The finite element model, to be useful, only needs to represent the primary contributors to the structural behavior of the vaults. Since the ribbed vaults absorb the majority of the energy induced by the impact hammer strike, the adjacent walls and surcharge are not modeled explicitly, but rather replaced with appropriate boundary conditions. The simplified version of the finite element model, which is used to generate transient vibration response, can be seen in Fig. 9.

The developed finite element model relies on many simplifying assumptions, such as the mesh discretization and selection of constitutive models. It is important to assess whether these modeling choices are appropriate for the purposes of this study. The finite element model developed for this study underwent a thorough Verification and Validation (V&V) process. Results of this extensive V&V study are documented by Atamturktur [26], who quantifies the prediction accuracy relative to vibration response measurements and demonstrates the appropriateness of the aforementioned assumptions. The V&V process employed in this study is similar to the one applied to the Washington National Cathedral by Atamturktur et al. [27]. An overview of finite element model calibration and validation studies, as applied to large-scale historic masonry monuments, has been written by Atamturktur and Laman [28].

To investigate the changes in the vibration response, four different finite element models are developed to represent (1) the undamaged vault, (2) the damaged vault, (3) the undamaged vault with variation in geometry and (4) the undamaged vault with variation in material. The finite element model of the *undamaged* vault is built with horizontal restraints from the nave walls, while the finite element model of the *damaged* vault is left free to translate horizontally at the peripheries of the walls (Fig. 9). Next, the material properties and geometry of the undamaged model are altered according to their estimated natural variability.

5.2. Finite element simulations

The finite element model is executed to mimic the experimental set-up as closely as possible and the vibration response of the vault is simulated for the four aforementioned scenarios: (1) the undamaged vault, (2) the damaged vault, (3) the undamaged vault with variation in geometry and (4) the undamaged vault with variation in material. The vibration response of interest is the transient response of 39 selected nodes due to an impact force applied at four separate excitation locations.

First, we look at the changes in the frequency response functions (FRF) of the vaults. Simulated FRFs estimate the vibration response of the structure due to a given force within the frequency domain. The FRF can be conveniently constructed from the simulated transient response of the vaults by taking the ratio of the Fast Fourier Transform (FFT) of the measured acceleration response and forcing functions. Fig. 10 compares the driving point FRFs measured at the crown of the vault. Looking at the amplitudes of these FRFs, while the change in the FRF amplitudes between the damaged and undamaged vault is about three to four fold, the change in the FRF amplitudes due to the 6% variation in geometry and 10% variation in material properties is consistently less than 20%.

Next, the autoregressive support vector machine (AR-SVM) approach is applied to these simulated vibration responses. Theoretical background of the AR-SVM approach will be provided later in Section 9. The average absolute residual errors of the AR-SVM model fit [29] is calculated for each scenario (Table 2). Table 2 shows that the geometric variation case is restricted to two sensors, since the geometric distortion requires a change of sensor locations in the model. Table 2 also includes the average absolute difference between each scenario and the undamaged case. From these simulations, it is obvious that the AR-SVM model best fits the material and geometric cases. Although the AR-SVM model fits for the material and geometric cases are not perfect, they are far superior to the AR-SVM model fit for the damaged case.

From this comparison, we can determine that separation between the nave walls and vaults is the most significant contributor

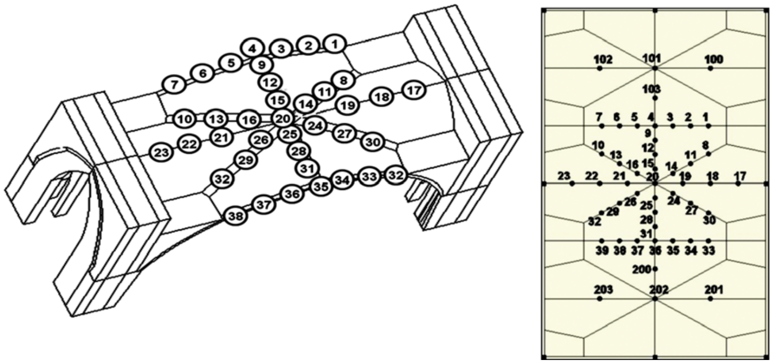

Fig. 11. A total of 39 measurement points are selected according to the mode shape predictions of the preliminary model. The excitation points are #20, #18, #12 and #11.

Table 2
Average absolute residuals of AR-SVM fit under several scenarios.

Sensor	Undamaged	Damage	Materials	Geometry
22	0.0463	0.8221	0.1317	N/A
18	0.0478	0.7950	0.1279	N/A
4	0.0974	1.1113	0.0769	0.0795
35	0.0930	0.8115	0.1032	0.0805
Average diff.	N/A	0.8138	0.0491	0.0152

to changes in AR-SVM damage indicators. With this determination comes a caveat. The contribution of each source of uncertainty to the lack of model fit is not necessarily linear, and hence combinations of scenarios may not lead to an additive change in model goodness-of-fit. However, the sources of uncertainty used in the simulations are representative of the most extreme cases for the material and geometric variations as indicated by our site surveys and field testing.

6. Vibration test campaign

Unlike modern civil structures, historic masonry structures pose unique challenges due to the behavior of their distinct structural systems. In a masonry system, the rigidity of a connection between two structural elements is affected by several factors, such as the contact pressure, surface friction, elastic behavior of each stone unit and mortar joint and existing cracks and hinges. Compared to contemporary reinforced concrete and steel structures, the inter-element connectivity is often more flexible in masonry systems. As a result, local vibration modes tend to be more pronounced than global modes. This makes the vibration response dependent upon the location of the excitation force. Also, the amplitude of the excitation alters the behavior of connections between structural elements and thus alters the response of the system. Compounding these difficulties is the presence of high dissipative forces inherent in masonry assemblies that complicate the identification of low amplitude global modes in the spectra.

These issues common to masonry systems are only a few of the hindrances to successful vibration testing of historic structures. Practical issues of testing a large-scale vaulted church, such as Beverley Minster, affect the outcome as well. Practical issues may include, but are not limited to, limited access to the site and attaching testing devices to the curved geometry of vaulted systems. A comprehensive discussion of the particulars and practicalities of *in situ* vibration testing procedures for complex vaulted masonry structures can be found in [30].

In the following sections, specifications of the vibration test are discussed, then a brief summary of quality checks implemented to verify the linearity and reciprocity of measurements is provided.

Specifications of the vibration test: The test is conducted in four phases, during which 16 accelerometers are moved to cover

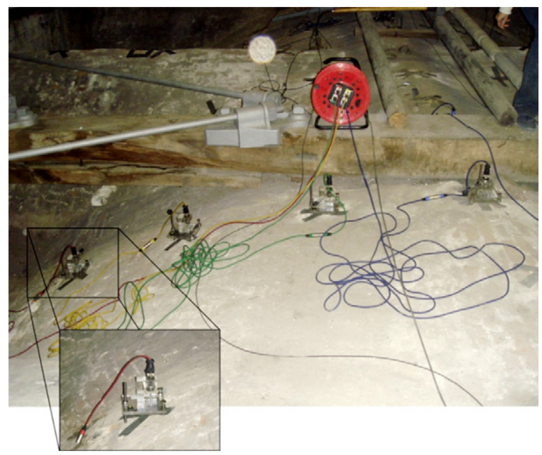

Fig. 12. The uni-axial accelerometers are placed on the curved vault surface with the help of adjustable mounting cases such that their axes remain vertical.

the measurement grid. The distribution of measurement points is determined according to preliminary finite element simulations of the vaults. The preliminary finite element model predicts modes as primarily composed of symmetric movements of the diagonal and orthogonal axes, as well as the crown. Based on this observation, a total of 39 measurement points are located at every quarter length on the main axis of the quadripartite vaults. To observe the interaction between the adjacent vaults, an additional eight measurement points are located on the two adjacent vaults (Fig. 11).

The transducers used in this study are Q-Flex QA 750 model force balance accelerometers, manufactured by Honeywell Inc. The accelerometers have a nominal sensitivity of 1.5 mA/g, which results in a voltage-sensitivity of 7.5 V/g, when dropped over a 5 kΩ resistor. They maintain a frequency range of 0–300 Hz and an amplitude range of ± 30 g. Due to the steep, curved surfaces of the vaults, mounting cases with adjustable screws are used to achieve precise alignment. The unidirectional accelerometers are mounted on the vault surface, such that they achieve a vertical axis of alignment (Fig. 12).

Impact hammers, shakers, and heel-drops are common controlled excitation devices used for traditional modal analysis. Among these excitation devices, the portability of impact hammers makes them preferable for this study. The mass of the hammer and stiffness of its tip define the frequency content of the excitation. The vaults are excited through the impact of a 5803A model sledgehammer (12 lb head), manufactured by Dytran Instruments, Inc. To broaden the impact duration and induce low-frequency vibration, the softest hammer tip is preferred.

Fig. 13. The hammer operator exciting the pre-determined excitation points with the sledgehammer. Maintaining a relatively consistent excitation level is one of the keys for the success of hammer excitation.

Although the acceleration response of the vault is measured solely in the vertical direction, the hammer excitation is applied perpendicularly to the vault surface; thus, modes with predominantly horizontal movement and less dominant vertical movement are also excited. As long as the accelerometers can detect the vertical acceleration of the vault, then the vertical components of these modes are also identifiable. *A priori* finite element model simulations and past experience gained from the testing of similar structures reveal that the first few modes involve deformation shapes that concentrate on the crown and the midpoints of the orthogonal and diagonal axes. As the resonant frequencies increase, the vault webs become more involved in the deformation and the mode shapes become complicated. To excite the majority of lower modes with detectable amplitudes, the use of four excitation points, situated at the centers of the main axis and crown, is most effective.

A significant drawback of hammer excitation is the inability to replicate the impacts with consistent excitation force (Fig. 13). Although the hammer operator swung the hammer as consistently as possible, the excitation force during the experiments varied between 1800 and 2200 N throughout the tests. To reduce the degrading effects from this uncontrolled variation of impact force, as well as from ambient noise, five impacts are performed and the responses are averaged for each excitation location.

Data acquisition is conducted using a 24 channel, 24 bit Data Physics Mobilyzer II spectrum analyzer. The upper frequency limit is 100 Hz and the data capture time is 16 s. This data configuration yields a 0.0625 Hz frequency resolution and 0.005 s time resolution. As the amplitude of the response diminishes within the data capture time frame, a rectangular window function is used for both impact and response signals.

Since masonry systems have inherently high damping compared to steel or reinforced concrete structures, artificial damping introduced by an exponential window can result in lower amplitude global modes being overpowered by adjacent, higher amplitude local modes. For this reason, the exponential window is avoided during data acquisition. However, during the modal extraction stage, a low order exponential window is used to clean the degrading effects of extraneous excitation.

A typical time history measurement of the hammer impulse and of the associated vault response is shown in Fig. 14. These time domain measurements are readily converted into the frequency domain by the spectrum analyzer. In the frequency domain, measured vault response is normalized with respect to the corresponding hammer impulse. This normalization process yields the experimental counterpart of the previously introduced frequency response function (FRF). Moreover, the coherence functions are obtained from the five repeated measurements. Coherence functions

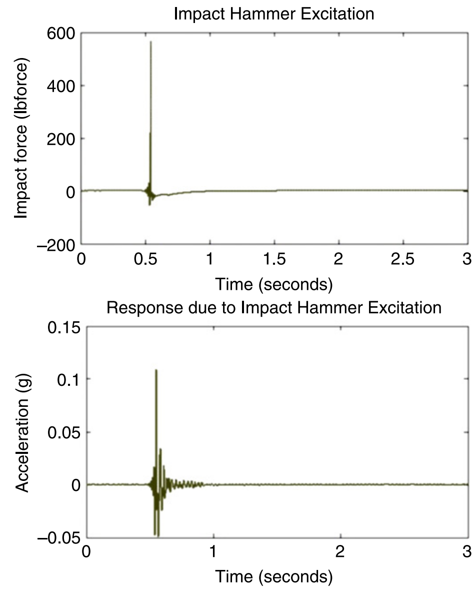

Fig. 14. Examples of measured signals: (top) hammer impact force in the time domain and (bottom) the response of the vaults due to the hammer impact.

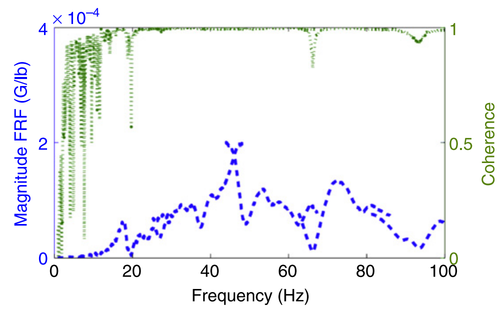

Fig. 15. A representative FRF and coherence function obtained from the undamaged vaults.

assess the extent to which the input and output signals maintain a linear relationship and thus conveniently determine the quality of measurements. Representative FRF and coherence plots are provided in Fig. 15. Given the aforementioned difficulties in performing vibration tests on large-scale historic masonry monuments, the measured coherences are deemed to be sufficiently high to verify that the structure responds within the linear regime.

From the coherence functions, the uncertainty in magnitude and phase FRFs can be computed using Bendat and Piersol's [31] formulation. This approach assumes that FRF variability is random and follows a Gaussian distribution. The variability of FRFs with one standard deviation is shown in Fig. 16. Fig. 16 was developed using driving point measurements—the excitation and measurement of the same point. The standard deviation is derived using two driving point FRFs collected from points 12 and 20. Fig. 16 illustrates that while the standard deviation of the FRFs obtained from undamaged and damaged vaults are comparable, the FRF amplitudes are generally higher for the damaged vault than they are for the undamaged vault. It is plausible that the damage introduced high amplitude, local modes at a frequency higher than 100 Hz. From Fig. 16, it is evident that such a high amplitude local mode is not present for the undamaged case.

Quality checks: Standard experimental modal analysis applications assume that the test specimen exhibits linearity and reciprocity.

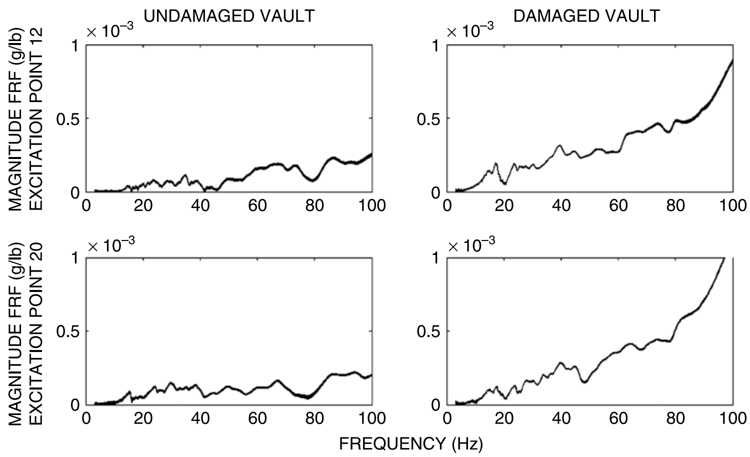

Fig. 16. The frequency response function of the damaged vault tends to have higher amplitudes compared to the undamaged vault, especially at higher frequencies.

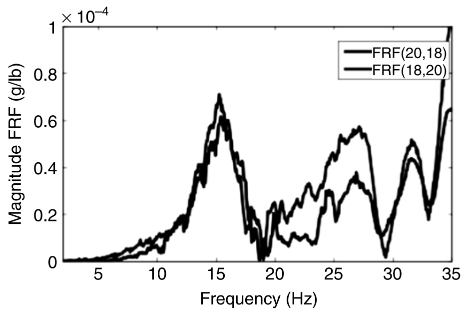

Fig. 17. The reciprocity check completed between measurement point 18 and 20 for the damaged vault.

Reynolds and Pavic [32] provide a discussion of quality assurance of test data obtained from civil engineering systems. As part of the quality assurance procedure, reciprocity checks are completed to confirm the linear behavior of the vaults under the excitation forces (Fig. 17). However, the judgment of the analyst is necessary for determining the acceptability of the deviations in the reciprocity check. Once again, considering the inherent variations in both the testing procedure and the tested structure, the correlation obtained between FRF(18, 20) and FRF(20,18) of Fig. 17 is deemed acceptable. Also, because the peaks of these two FRFs remain nearly unchanged, the identification of modal parameters is minimally affected by the presence of the deviations.

7. Evaluation in the modal domain

In Section 5, the effect of geometric and material variability on the vibration response was quantified, through numerical simulation, to show that inherent vault-to-vault variability changes the vibration response of the vaults to a lesser extent than structural damage. Since our modeling assumptions are conservative and the FE models have undergone rigorous V&V, we conclude that analyzing measurements collected on different vaults is not detrimental to answering the main question: can the presence of damage be inferred from the vibration response? An answer is first attempted in the modal domain. Sections 8 and 9 then discuss evaluations performed in the frequency and time domains, respectively.

The complex geometry of the vaults, along with the particularities of masonry construction, often yields an abundance of complex, closely spaced modes. As evidenced by the FRF given in Fig. 15, approximately twenty modes of significant amplitude are present between 0 and 20 Hz. However, extracting reliable modal parameters from these high modal density measurements is a difficult endeavor, especially for modes with low participation factors. Moreover, the unique challenges of hammer testing, such as poor signal-to-noise ratio and high crest factor, further challenge the accurate extraction of higher-order modal parameters. Thus, the number of modes that can be used during the comparison of undamaged and damaged vaults is typically limited. On the other hand, operating on a limited number of modes is not a significant drawback. As the mode order increases, the mode shapes become more and more dominated by local response and highly sensitive to the excitation location. Thus, higher-order modes typically do not contain information regarding the global damage. In the present study, the comparisons of the undamaged and damaged vaults through modal parameters are limited to the first ten natural frequencies and mode shapes. The estimation of damping ratios is known to be significantly less accurate, when compared to the natural frequencies and mode shapes, so the damping ratios are not incorporated in the comparison.

In the present study, modal extraction is conducted using the ME'scope Version 4.0 software, developed by Vibrant Technology, Inc., with a multiple-reference, global curve-fitting algorithm that combines FRF measurement data from multiple excitation locations. Once the modal parameters for damaged and undamaged states are identified from the FRF measurements, the differences between natural frequencies and mode shapes are quantified.

Table 3 presents the differences in natural frequencies of the undamaged and damaged vault. To be statistically significant, a change in natural frequency due to damage should exceed, by a factor of two or more, the level of experimental variability. This is not observed in Table 3, since the natural frequencies are shifted by a maximum of 0.14 Hz. This magnitude of frequency shift is similar to the experimental variability obtained by replicating the measurements on a similar Gothic church, Washington National Cathedral [26]. The frequency variations can potentially be attributed to the perturbation introduced by the presence of a hammer operator. In contrast with earlier, successful studies (see for instance [4,10]) that correlated damage with reduction in natural frequencies, the modal properties associated with lower-frequency global modes of the Minster vaults are observed to be insensitive to the existence of wall–vault separation.

Table 3 also includes a correlation metric used to compare the mode shapes of the two prototype vaults. In this study, the mode shape vectors include the motion of twenty-seven measurement points relative to each other and thus have a higher dimensionality than natural frequencies. Therefore, the Modal

Table 3
The modal parameters identified from damaged and undamaged vaults.

Mode #	Undamaged vault Frequency (Hz)	Damaged vault Frequency (Hz)	Δf (Hz)	Mode shape correlation MAC (Unitless)
1	3.38	3.38	0	0.936
2	3.87	3.87	0	0.813
3	4.85	4.92	0.07	0.927
4	5.62	5.72	0.04	0.493
5	6.36	6.34	0.02	0.371
6	7.77	7.63	0.14	0.464
7	8.59	8.58	0.01	0.245
8	8.99	9.00	0.01	0.742
9	9.39	9.41	0.02	0.509
10	9.96	10.0	0.04	0.658

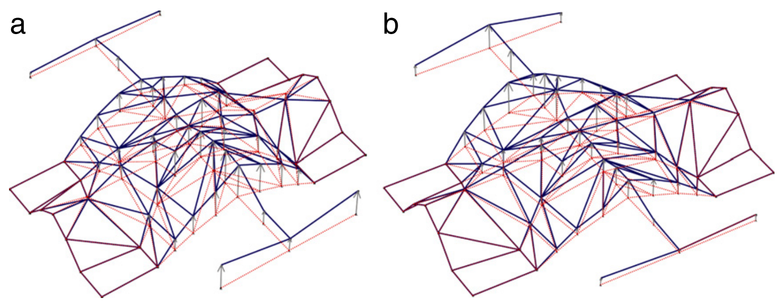

Fig. 18. The first mode shape of (a) undamaged vault, (b) damaged vault.

Assurance Criterion (MAC) is used to obtain a lower-dimensional metric to compare the mode shapes obtained from the damaged and undamaged vaults. A MAC value of 1.0 represents a perfect correlation between two mode shape vectors, while 0.0 indicates two orthogonal mode shape vectors, i.e. uncorrelated mode shapes. Due to the complexity of *in situ* experiments, MAC values of 80% or higher are considered satisfactory for the purposes of this study. Fig. 18 provides a visual comparison of the first modes of the two prototype vaults, which yield a MAC of higher than 90%. In Table 3, mode shape vectors are shown to yield good correlation for the first three modes, while higher-order modes exhibit less correlation. This observation is in agreement with an earlier study by Ramos [10], which demonstrated the insensitivity of mode shapes to structural damage. However, the higher-order, uncorrelated mode shapes are possible indicators of differences between two structural conditions, i.e. wall–vault separations. This statement assumes that the system identification is completed with sufficient accuracy.

Our initial hypothesis was that structural damage manifests itself as a change in the natural frequency of the low-order resonances. However, as seen in Table 3, the first three modal parameters remain unchanged irrespective of the damage state of the vaults. The similarities in the first three global modes in the two test structures supports our assumption that these two vaults indeed have comparable structural properties, such as boundary conditions and material properties.

8. Evaluation in the frequency domain

Modal parameters provide physically meaningful and convenient features for the comparison of two datasets. However, when using modal parameters the comparative analysis may suffer from (1) low feature dimensionality and (2) incomplete measurements. In this section, the direct comparison of FRFs is used as a convenient, higher dimension alternative to comparing modal parameters. Also, the use of FRFs eliminates the use of curve-fitting algorithms to extract modal parameters.

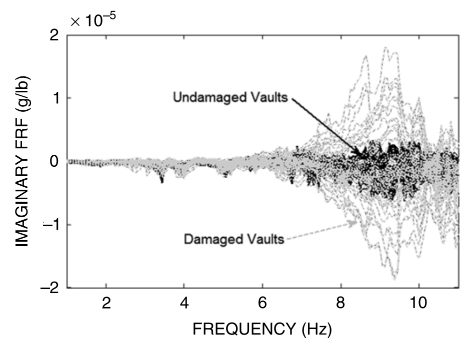

Fig. 19. The imaginary component of the FRF conveys the relative deformations of the measurement points, which are observed to be comparable for the damaged (red dashed) and undamaged (blue solid) vault for frequencies below 7 Hz. Frequencies higher than 7 Hz have significantly higher amplitudes for the damaged vaults.

Overlaying FRFs obtained from the damaged and undamaged vaults gives a visual indication of change between the structures. As Fig. 19 shows, the FRFs obtained from the two vaults agree relatively well up to 7 Hz. For higher frequencies, the FRF obtained from the undamaged vault has significantly lower amplitudes than the FRF obtained from damaged vault. The same trend is observed consistently in all FRFs (Fig. 19) and is in agreement with the simulated response of the vaults as given in Fig. 10. Through this visual assessment, the FRFs yield a clear indication that a change between the two structural systems has occurred. This observation is consistent with the FRFs simulated by the FE model (see Section 4), as indicated in Fig. 10. If the onset of damage introduces nonlinearity to a predominantly linear system, then coherence functions can be used as damage indicators. A typical coherence function, corresponding to the driving point measurement at the crown of the undamaged vault, can be seen in Fig. 20. The coherence functions of the damaged vaults are observed to be lower than those of the undamaged vaults, possibly due to system damage amplifying the nonlinearity of the vibration response.

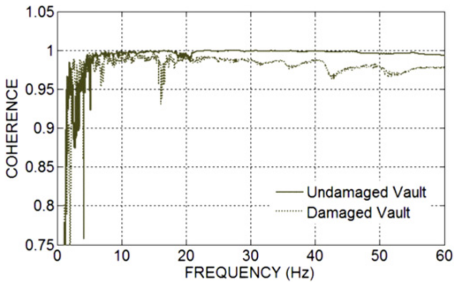

Fig. 20. The coherence plot indicates the linear relationship between the input force and output response. The damaged vault coherence plot shows a reduction in this linear relationship.

Using the coherence functions in addition to FRFs may provide a diagnostic that, while being insensitive to environmental and testing variability, correlates well with the presence of structural damage.

9. Evaluation in the time domain

Comparisons of FRFs are useful as they filter out the undesired noise from the measurements and provide smoothened information about system behavior over a wide frequency range. However, like modal parameters, obtaining an FRF is based on the assumption of linearity. This assumption may become a problem, since damage may introduce nonlinear effects into the system that cannot be captured adequately by a linear model [33]. Given a nonlinear response, an FRF provides a smeared representation of the nonlinear effects. Time domain methods, however, may offer higher fidelity in representing nonlinearities and may have better success in detecting structural damage.

Regression models are applied in the time domain with model residuals acting as the damage indicators. In addition to providing a smoothing effect to the raw measurements, this procedure offers the advantage of defining scalar-valued features that lower the dimensionality of the time series. Specifically, an autoregressive (AR) model is best-fitted to a time domain signal known (or assumed) to be collected on a damage-free structure. The degree of goodness-of-fit of the AR representation is used as the damage-sensitive indicator. Model residuals, defined as the difference between predictions of the AR model and the experimental data, are monitored for statistically significant changes assumed to be caused by damage. An AR model of the kth sensor with p autoregressive terms, AR(p), is expressed as:

$$x_t^k = \sum_{j=1}^{p} \beta_j^k \cdot x_{t-j}^k + \varepsilon_t^k \tag{1}$$

where x_t^k is the measured signal from sensor k at discrete time t, β_j^k are the AR coefficients or model parameters, and ε_t^k is an unobservable noise term.

It can be observed in Eq. (1) that an AR model best fits each sample of the time domain signal with a linear combination of the previous p samples. While autoregressive models work particularly well when modeling the response of linear, time-invariant systems, systems exhibiting nonlinearity in their initial state or time-varying responses (such as those from hammer-excitation experiments) can result in mediocre goodness-of-fit. Such poor model fit could, in turn, feature low sensitivity to the onset of damage. To address this concern, and because it is well known that AR models do not always represent transient data well, we turn to support vector regression methods [29].

For autoregressive support vector machines (AR-SVM), the model takes the form

$$x_t^k = \sum_{j=p+1}^{t_0} \beta_j f(\mathbf{x}_{j-p:j-1}^k, \mathbf{x}_{t-p:t-1}^k) + \varepsilon_t^k \tag{2}$$

where the vector $\{x_{t-p}^k, \ldots, x_{t-1}^k\}$ is denoted as $x_{t-p:t-1}^k$ for sensor k. Also, f is a kernel function capable of modeling nonlinear relationships and t_0 is the length of the undamaged time domain signal used to train the model. With the appropriate choice of parameters, including the kernel function f, its associated parameters, and the training set length, an AR-SVM model is able to represent any nonlinear relationship between the current time point, x_t^k, and the p previous time points, $x_{t-p:t-1}^k$. Highly adaptable and generalizable, it has been established that this approach performs well in high-dimensional spaces and outperforms conventional AR models when applied to transient signals. Even though they can be seen as being similar to neural networks proposed for SHM [14], AR-SVM models only require a simple quadratic optimization for training. Despite their simplicity relative to neural networks, these models achieve equivalent, if not superior, prediction accuracy as demonstrated by Scholkopf et al. [34].

To ensure that signals from both damaged and undamaged vaults are comparable, they are first normalized by the impact level of the hammer strike. Further, to simplify the choice of model parameters, signals to be compared are scaled by the standard deviation of the undamaged signal [29]. An exponential smoothing window is applied to attenuate any noise artifacts. The procedure implemented to compare the vibration responses of damaged and undamaged vaults follows the steps outlined next. AR-SVM models are first trained on time series collected for the undamaged vault and one model is developed for each sensor location. Next, the trained models are used to predict signals for both undamaged and damaged vaults. To ensure that the method is insensitive to vault-to-vault, experimental, and environmental variability, the symmetry of the vault and roving sensor placement are exploited. Each AR-SVM model is trained and tested on two separate, but related, undamaged signals opposite from each other with respect to the excitation location as shown in Fig. 11. For example, the AR-SVM model developed for sensor 203 of the undamaged vault is subsequently tested on sensor 102 of both the undamaged and damaged vault. Testing predictions of the AR-SVM models with time series, other than those used to train the models, guards against over-fitting. It also helps to develop diagnostics of structural damage that, because they are based on statistics of lack-of-fit residual errors, account for the environmental variability.

This procedure is repeated for all 46 sensor locations, excluding the crown for which a complementary sensor location does not exist. Examples of time series and AR-SVM model fits for both the undamaged and damaged vault are shown in Fig. 21. It can be observed that the model fit to the undamaged vault data, although not perfect, is far superior to the model fit to the data from the damaged vault. Examining the lack-of-fit residuals of AR-SVM predictions for all sensor locations reveals that the undamaged case has a significantly stronger goodness-of-fit. This can be quantified using, for example, statistics from the lack-of-fit residuals. The average absolute value of residual error is plotted for each sensor location in Fig. 22, sorted according to values of the damaged vault. A t-test statistic, which tests for equal means between two normally distributed samples, indicates a systematic difference between the two datasets with a p-value below 10^{-15}. Generally p-values below 0.01 are considered to correlate with strong evidence. Therefore, we conclude from both graphical observation and statistical testing that the AR-SVM models provide a significantly better fit to the undamaged vault. Because the training of AR-SVM models included a cross-validation step to prevent over-fitting and to improve prediction under various sources of variability, we conclude that the systematic lack-of-fit observed when applied to signals collected on the damaged vault come from structural damage.

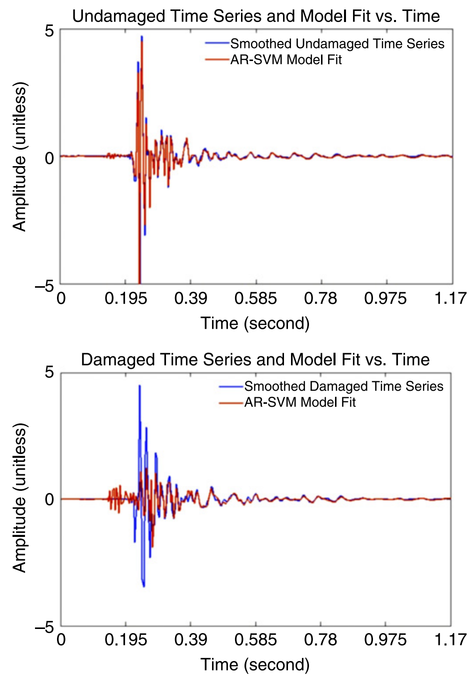

Fig. 21. Comparison of AR-SVM fit to normalized transient impact data in (top) undamaged and (bottom) damaged cases. Average absolute residuals for this sensor in the undamaged and damaged cases are 0.0412 and 0.1308 respectively, indicating significantly improved model fit to the undamaged case.

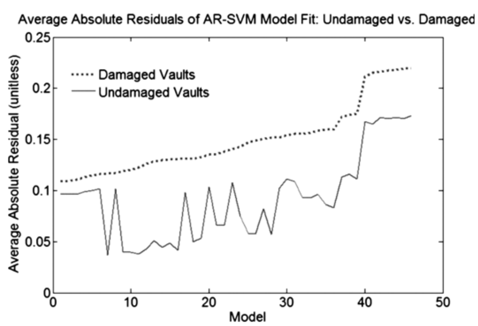

Fig. 22. Average absolute values of lack-of-fit residuals of AR-SVM predictions at 46 sensor locations for the undamaged and damaged cases.

10. Conclusions

The nave walls supporting the vaults of Beverley Minster have been steadily moving for eight centuries. Due to this movement, the ten originally equivalent masonry vaults have undergone a non-uniform damage pattern. This has led to two outcomes: (1) severe separation of the vaults from the wall and (2) geometric distortions of the vaults. Both of these outcomes caused a reduction in structural strength of the vaults.

When damage occurs internally, as in the case of Beverley Minster vaults, visual identification of impending failure becomes difficult. This difficulty is often the case for tensile problems in masonry monuments. In these situations, damage detection techniques based on vibration characteristics provide a particularly advantageous monitoring and assessment tool. If internal damage in a masonry structure is ignored completely, the load-carrying capability of the structure may be compromised, which may lead to a collapse that occurs without warning. Aside from the obvious economic and life-safety implications, the cultural and historical value of historic monuments adds significance to the development of quantifiable methods capable of assessing the adverse effects of support settlements. By implementing early diagnosis, the direct cost of repair can be reduced significantly.

The datasets analyzed in the present study are force and acceleration time series collected through modal tests performed on damaged and undamaged vaults. Given the specificities of the Beverley Minster, the hammer impact technique was deemed the most appropriate excitation. Because the raw datasets collected are transient time series, analysis can potentially take place in the time, frequency, or modal domain. Analysis in the time domain offers the advantage of processing the most general-purpose signals, but requires efforts to reduce dimensionality and eliminate potential artifacts that contaminate measurements and could mistakenly be interpreted as a manifestation of structural damage. On the other extreme, analysis in the modal domain offers the advantage of averaging, smoothing, and data compression, at the expense of relying on strong assumptions, such as stationary, reciprocal, and linear behavior.

First starting with raw measurements, Auto-Regressive Support Vector Machine (AR-SVM) models show great success in detecting the difference between the two vault conditions. In particular, AR-SVM models trained with signals from the undamaged vault are able to accurately fit measurements from the adjacent, undamaged vault, but are incapable of correctly modeling signals from the damaged state. The second option investigated is to analyze measurements in the frequency domain. The imaginary parts of Frequency Response Functions (FRFs) indicate that local modes of the damaged vault provide significantly higher amplitudes that those of the undamaged vault. This observation is consistent with the hypothesis that the excitation causes the damaged vault to deflect more than the undamaged one, a clear indication of loss of dynamic stiffness. The direct comparison of FRF amplitudes and coherence functions may define a convenient and damage-sensitive tool for future, in situ monitoring of historic masonry structures.

Proceeding to the most processed form of data analysis, modal frequencies and mode shapes are estimated next. Contrary to expectation, the first three natural frequencies of the damaged vault are found to be substantially similar to those of the undamaged vault. Likewise, the first three mode shape vectors are mostly unchanged even though some of the higher-order modes are difficult to correlate. This analysis is inconclusive and sheds doubt on the effectiveness of modal-based techniques when applied to realistic datasets.

Contrary to the prevalent use of modal-based methods for structural health monitoring, our overall conclusion is that time domain analysis may provide a reliable diagnosis, as long as steps are taken to ensure that the effects of structural damage can be separated from those of environmental variability. Finite element simulations demonstrate that our AR-SVM methodology, while able to detect the difference in vibration response due to the presence of damage, can be made insensitive to various sources of uncontrolled, geometric, and material variability. Though these results point to a clear potential of time domain methods for damage detection applied to historic masonry monuments, their effectiveness when dealing with less severe levels of damage remains to be investigated.

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