

Single sample binomial tests and their properties

a. Consider the case of X distributed as $Bi(n, \pi)$. Derive expressions for the Wald test and the Score test (un-squared version) as functions of $p = X/n$ for the one-sided hypothesis $H_0 : \pi = \pi_0$ versus $H_1 : \pi > \pi_0$. Do the same for the log-likelihood test, but in terms of $o = X$ and $e = E\{X\}$.

b. Examine power and type I error rates the above tests for $\pi_0 = .1, .25$ and $.50$ for $n = 10, 25$ and 50 using exact calculations. (Hint: Obtaining exact algebraic representations for the type I and type II error rates is somewhat messy. Computer calculations are greatly simplified by simply tabulating binomial probabilities over the relevant values of π and n and representing rejection regions by indicator vectors of length n which can then be used to pick out the relevant probabilities for the type II errors)