

MDSC 643.01, Biostatistics I

Brant, Fall 2002

Comparing Proportions in Three or More Samples

We have used 2×2 tables as a basis for comparing two proportions, leading to use of the chi-square test (or Fisher's Exact test, depending on the expected counts). If we have binary observations from $k > 2$ samples, we can consider testing the global hypothesis, $H_0: p_1 = p_2 = \dots = p_k$, which can also be considered as a hypothesis of no association between the grouping factor and the "outcome" variable.

Example:

Here is a table (of frequencies and row percentages) comparing transfusion rates for orthopedic surgery at three surgical centers A, B, and C.

Hospital	Transfusion		Total
	No	Yes	
A	328 87.47	47 12.53	375 100.00
B	114 86.36	18 13.64	132 100.00
C	350 77.78	100 22.22	450 100.00
Total	792 82.76	165 17.24	957 100.00

To examine differences, we could make pairwise comparisons based on the sample proportions (Query: what are the relevant \hat{p}_i 's and n_i 's?).

However, the same logic behind the chi-square test for a 2×2 table applies

here, leading to the use of $X^2 = \sum \frac{(O - E)^2}{E}$ where the O 's are the observed

frequencies in the inner table and the E 's are expected values defined by

$$E = \frac{(\text{Row Total})(\text{Column Total})}{N}$$

If H_0 holds X^2 follows a chi-squared distribution with $k - 1$ degrees of freedom.

example:

$r \times c$ tables

If one has a cross-tabulation based on two factors, one with r levels and a second with c levels, then the result is an $r \times c$ table. The chi-square approach above can be applied to produce a global test for association between the two factors. In this case the relevant degrees of freedom are $(r-1) \times (c-1)$.

Example: a study relating student alcohol use to parental use.
(percentages are column percentages).

Student's Alcohol Use	Parental Alcohol Use			Total
	Never	Occasional	Regular	
Never	141 60.00	68 41.72	17 36.17	226 50.79
Occasional	54 22.98	44 26.99	11 23.40	109 24.49
Regular	40 17.02	51 31.29	19 40.43	110 24.72
Total	235 100.00	163 100.00	47 100.00	445 100.00