

Probability

Probability - provides a basis for making "logical" arguments based on uncertain facts

Basic logic:

If Hypothesis A implies that Event B can't happen and B happens then A must be false.

Probabilistic logic:

If Hypothesis A implies that Event B has low probability and B happens then A is less plausible

Definition of probability:

Begin with some event A defined in context of underlying scenario
e.g. Probability of Heads in coin toss: fair coin? method of tossing?

Probability of A = p , $0 \leq p \leq 1$, which describes?

- likelihood of A - long term relative frequency?
- perceived likelihood? (as characterized by betting behaviour?)

Determining Probabilities

Assumed probabilities (axiomatic?)

e.g. Physical argument, fair coin, $\Pr\{\text{Heads}\} = .5$
or $\Pr\{\text{Randomly selecting } M\} = \text{fraction of } M$

Calculating probabilities - rules are same under either definition of probability

e.g. background - Genetic Risk Marker - GM, Family history - FH

	FH=yes	FH=no
GM=yes	1000000	1000000
GM=no	2000000	96000000

Scenario: Randomly select an individual, Events GM=yes, FH=no, etc.

$\Pr\{\text{GM=yes}\}$, $\Pr\{\text{FH=no}\} = ?$

Complement of an event: $\Pr\{\text{GM=no}\} = 1 - \Pr\{\text{GM=yes}\} =$

Intersection of two events: $\Pr\{\text{GM=no AND FH=yes}\} =$

Union of two events: $\Pr\{\text{Either GM=yes OR FH=yes}\} =$

“Calculus of Probability” - consider events E1 and E2

Additive Rule for OR: $\Pr\{\text{Either E1 OR E2}\} =$

Special Case: E1, E2 mutually exclusive (E1 \rightarrow not E2, vice versa)

Rules for AND: use conditional probability corresponding to “restriction” of original “scenario”

e.g. What is $\Pr\{\text{FH=yes}\}$ given GM=yes : $\Pr\{\text{FH=yes} \mid \text{GM=yes}\}$

$\Pr\{\text{GM=yes} \mid \text{FH=yes}\} =$

$\Pr\{\text{FH=yes} \mid \text{GM=no}\} =$

Multiplicative Rule: $\Pr\{\text{E1 AND E2}\} = \Pr\{\text{E1}\} \times \Pr\{\text{E2} \mid \text{E1}\}$
 $= \Pr\{\text{E2}\} \times \Pr\{\text{E1} \mid \text{E2}\}$

Independent events - If $\Pr\{\text{E2} \mid \text{E1}\} = \Pr\{\text{E2}\}$ E1, E2 said to be independent.

Simplified multiplication rule - $\Pr\{\text{E1 AND E2}\} = \Pr\{\text{E1}\} \times \Pr\{\text{E2}\}$